Spring 2020 Math 245 Exam 2

Please read the following directions:

Please write legibly, with plenty of white space. Please fill out the box above as legibly as possible. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 10:00 and will end at 10:50; pace yourself accordingly. Please remain quiet to ensure a good test environment for others. Good luck!

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. Proof by Cases Theorem

b. Nonconstructive Existence Theorem

c. Proof by Reindexed Induction

Problem 2. Carefully define the following terms:

a. Proof by Minimum Element Induction Theorem

b. well-ordered by $<$

c. big O

Problem 3. Let $x \in \mathbb{R}$. Use cases to prove that $|x-2|+|x-5| \geq 3$.

Problem 4. Prove that $\forall x \in \mathbb{R},\ \lfloor -x \rfloor = -\lceil x \rceil$.

Problem 5. Use induction to prove that for all $n \in \mathbb{N}$, $\binom{2n}{n} \le 4^n$.

Problem 6. Solve the recurrence with initial conditions $a_0 = 1, a_1 = 4$ and relation $a_n = 3a_{n-1} - 2a_{n-2}$ $(n \ge 2)$.

Problem 7. Consider the nonstandard order $\prec$ on $\mathbb{Z}$ given by $0 \prec 1 \prec -1 \prec 2 \prec -2 \prec 3 \prec \cdots$. The smallest element is 0, the second smallest is 1. Find a formula for the n^{th} smallest element.

Problem 8. Consider the sequence $a_n = 3n^2 + 100n + 1$. Prove that $a_n = \Theta(n^2)$.

Problem 9. Prove that $\forall n \in \mathbb{N}_0$, the Fibonacci numbers F_n satisfy $F_n < 1.9^n$.

Problem 10. Find a recurrence relation for sequence T_n such that the Master Theorem would give $T_n = \Theta(\sqrt{n} \log n)$. Describe an algorithm that would satisfy your recurrence relation.